



Pearson

Mark Scheme (Results)

Summer 2017

Pearson Edexcel International A Level in
Statistics S3 (WST03/01)

edexcel 

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Summer 2017

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if **the candidate's response is not worthy** of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme **to a candidate's response, the team leader must be consulted.**
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M marks: method marks are awarded for 'knowing a method and attempting to apply it',** unless otherwise indicated.
 - **A marks:** Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B marks** are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \checkmark will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - \square The second mark is dependent on gaining the first mark
4. **All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to** indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
 7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme									Marks	
1. (a)	Parrot	A	B	C	D	E	F	G	H	M1 M1 A1 dM1; A1 [5]	
	Rank Age	3	6	2	1	7	8	4	5		
	Rank Breeder	5	6	4	1	8	7	2	3		
	For finding the difference between each of the ranks and evaluating $\hat{a}d^2$										
	$\hat{a}d^2 = 4 + 0 + 4 + 0 + 1 + 1 + 4 + 4 = 18$										
For use of the correct formula with their $\hat{a}d^2$											
$r_s = 1 - \frac{6(18)}{8(8^2 - 1)}; = 0.78571429...$											
$\frac{11}{14}$ or awrt 0.786									A1		
[5]											
(b)	$H_0: r = 0, H_1: r > 0$									Both hypotheses stated correctly	B1
	Critical Value = 0.8333 or CR: $r_s \geq 0.8333$									Critical value of 0.8333	B1
	Since $r_s = 0.7857...$ does not lie in the CR (or $0.7857... < 0.8333$), do not reject H_0									see notes	M1
	Either conclude that									A1ft	
	<ul style="list-style-type: none"> the breeder does not have the ability to correctly order parrots by age, after examining them. there is insufficient evidence that the breeder can correctly order parrots by age. 										
[4]											
Notes											
(a)	1 st M1	Attempt to rank for actual ages or breeder's estimates of ages. (At least 4 correct in either row - allow reverse rankings)									
	2 nd M1	Independent of 1 st M1 but these must be ranks.									
(b)	3 rd dM1	<i>is dependent on 1st M1</i> for use of $1 - \frac{6(18)}{8(8^2 - 1)}$ with their $\hat{a}d^2$.									
	1 st B1	Both hypotheses correct in terms of r or r_s .									
	2 nd B1	Critical value of 0.8333									
	M1	For a correct statement relating their r_s ($ r_s < 1$) with their c.v. where $ \text{their c.v.} < 1$									
	A1ft	For a contextualised comment which is accepting H_0 , which must mention "breeder", "order", "parrots", which conveys the idea that the breeder cannot order them correctly. All previous marks in part (b) must have been scored to award this one.									
Note	Follow through their r_s with 0.8333										
Note	Two-tailed test Applying a two-tailed test scores a maximum of B0B1M1A0 So Award SC B0B1 for $H_0: \rho = 0, H_1: \rho \neq 0$ followed by critical value $r_s = (\pm) 0.881$ and allow access to the M1 mark only.										

Question Number	Scheme				Marks																																
2.	H ₀ : There is no association between gender and (inspirational) message (independent) H ₁ : There is an association between gender and (inspirational) message (dependent)				B1																																
	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Expected</th> <th>A</th> <th>B</th> <th>C</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>Male</td> <td>27.106...</td> <td>41.373...</td> <td>38.52</td> <td>107</td> </tr> <tr> <td>Female</td> <td>29.893...</td> <td>45.626...</td> <td>42.48</td> <td>118</td> </tr> <tr> <td>Total</td> <td>57</td> <td>87</td> <td>81</td> <td>225</td> </tr> </tbody> </table>				Expected	A	B	C	Total	Male	27.106...	41.373...	38.52	107	Female	29.893...	45.626...	42.48	118	Total	57	87	81	225	Some attempt at (Row Total)(Column Total) (Grand Total) M1												
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Observed	Expected	$\frac{(O - E)^2}{E}$	$\frac{O^2}{E}$																																		
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Observed	Expected	$\frac{(O - E)^2}{E}$	$\frac{O^2}{E}$																																		
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	$X^2 = \sum \frac{(O - E)^2}{E} \text{ or } \sum \frac{O^2}{E} - 225 ; = \text{awrt } 3.27$				For applying either $\sum \frac{(O - E)^2}{E}$ or $\sum \frac{O^2}{E} - 225$ dM1 awrt 3.27 A1																																
	$n = (2 - 1)(3 - 1) = 2$				$n = 2$ B1																																
	$\chi^2_{(0.10)} = 4.605 \Rightarrow \text{CR: } X^2 \geq 4.605$				4.605 B1ft																																
	[does not lie in the CR/not significant/Do not reject H ₀]																																				
	Either conclude that <ul style="list-style-type: none"> • there is <u>insufficient evidence to support the headteacher's belief.</u> • there is <u>no association between gender and inspirational message.</u> (They are independent) 				A1																																
					[10]																																
					10																																
Notes																																					
1 st B1	For both hypotheses. Must mention "gender" <i>and</i> "message" oe at least once.																																				
1 st M1	Use of "relationship" or "correlation" or "connection" or "link" is B0.																																				
1 st A1	Can be implied by at least one correct E_i to 1 d.p.																																				
2 nd dM1	At least 5 expected frequencies correct awrt or trunc. 2 d.p. [No fractions]																																				
2 nd A1	Dependent on 1 st M1 for at least 2 correct terms or correct expressions with their E_i																																				
3 rd dM1	At least 5 correct terms to either 1 d.p. or awrt/trunc. 1.d.p. (may be implied by awrt 3.27)																																				
Note	Dependent on 2 nd M1 For applying either $\sum \frac{(O - E)^2}{E}$ or $\sum \frac{O^2}{E} - 225$																																				
Note	If awrt 3.27 is seen (from a calculator) without the expected frequencies stated then award special case M0A0M1A1M1A1.																																				
2 nd B1	$n = 2$. This mark can be implied by a correct critical value of 4.605																																				
3 rd B1ft	4.605 or ft their ν																																				
4 th A1	Dependent on 3 rd M1 and 4.605. A correct contextualised conclusion which is accepting H ₀ .																																				
Note	Must mention either "headteacher's belief" or "gender" <i>and</i> "message".																																				
Note	Contradictory statements score A0. E.g. "significant, do not reject H ₀ "																																				
Note	Condone "relationship" or "connection" here but not "correlation".																																				
Note	Hypotheses the wrong way round is A0																																				

Question Number	Scheme	Marks
3. (a)	$H_0 : m = 30 \quad H_1 : m \neq 30$	B1
	$z = \frac{28.2 - 30}{\frac{8.5}{\sqrt{75}}}; = -1.833936...$	M1;
	$\pm \frac{28.2 - 30}{\frac{8.5}{\sqrt{75}}}$ or equivalent. awrt <u>-1.83</u>	A1
	Two tailed c.v.'s $Z = \pm 1.6449$ or CR: $Z \leq -1.6449$ or $Z \geq 1.6449$ or p-value = awrt 0.033 or awrt $0.034 < 0.05$	B1
	[in the CR/significant/Reject H_0 /"[0.033, 0.034]" < 0.05]	
	Conclude either <ul style="list-style-type: none"> that the <u>mean age of gym customers is not 30 years.</u> that the <u>manager's claim is not correct.</u> 	A1
		[5]
(b)	\bar{X} is (approximately) <u>normally distributed</u>	B1
		[1]
(c)	Assumed $s^2 = S^2$ or variance of sample = variance of population.	B1
		[1]
		7
Notes		
(a)	1 st B1 Both hypotheses correct.	
	M1 For standardising with 28.2, 30 and $\frac{8.5}{\sqrt{75}}$ (or awrt 0.981) [Allow use of $8.5 \times \sqrt{\frac{74}{75}}$ (=awrt 8.44)]	
	2 nd B1 Critical value of -1.6449 (compatible with sign of their test statistic) or a correct probability comparison.	
	2 nd A1 Dependent on M1 scored for a correct contextualised comment which is rejecting H_0 which is based on their z-value and their critical value with compatible signs, where $1.64 \leq c.v. \leq 1.65$ Contradictory statements score final A0. E.g. "significant, do not reject H_0 ".	
	Alternative method for the "M1A1B1" marks: Let \bar{X}_c be the critical value of the sample mean.	
	$-1.6449 = \frac{\bar{X}_c - 30}{\frac{8.5}{\sqrt{75}}}$	M1: For $\frac{c - 30}{\frac{8.5}{\sqrt{75}}} = -1.6449 / -1.645 / -1.64 / -1.65$
	So $\bar{X}_c = 28.38883812...$	A1: $\bar{X}_c =$ awrt 28.4 B1: Critical value of -1.6449
Note	One tailed test SC: Applying a one-tailed test scores a maximum of B0M1A1B1A0 (Allow ± 1.2816 to score the 2 nd B1)	
(b)	Allow in words e.g. " sample mean is normally distributed"	
(c)	B1 Also allow $s = S$ or standard deviation of sample = standard deviation of population.	

Question Number	Scheme	Marks																																																																					
4. (a)	$\hat{\lambda} = \frac{0(3) + 1(13) + 2(14) + 3(15) + 4(10) + 5(8) + 6(8) + 7(6) + 8(3)}{80} \left\{ = \frac{280}{80} \right\} = 3.5^*$	B1cso *																																																																					
		[1]																																																																					
(b)	$r = 80 \cdot \frac{e^{-3.5}(3.5)^3}{3!} \{ = 17.26283752... \}$ or $r = 80 \cdot (0.5366 - 0.3208) \{ = 17.264 \}$ $s = 80 - (2.42 + 8.46 + 14.80 + \text{their } r + 15.10 + 10.57 + 6.17 + 3.08) \{ = 2.14 \text{ or } 2.13716... \}$ or $s = 80 \cdot (1 - 0.9733) \{ = 2.136 \}$	M1																																																																					
	$r = 17.26$ (2dp), $s = 2.14$ (2dp) At least one of either $r = \text{awrt } 17.26$ or $s = \text{awrt } 2.14$	A1																																																																					
		Both awrt $r = 17.26$ and awrt $s = 2.14$																																																																					
		[3]																																																																					
(c)	H_0 : Poisson distribution is a suitable model. H_1 : Poisson distribution is not a suitable model.	B1																																																																					
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	$n = 7 - 1 - 1 = 5$	B1ft																																																																					
	$\chi^2_5(0.05) = 11.070 \Rightarrow \text{CR: } X^2 \geq 11.070$	B1ft																																																																					
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Notes		
(a)	B1cso*	At least 2 non-zero products shown and divide by 80 to achieve 3.5*
(c)	1 st B1	Must have both hypotheses and mention Poisson at least once. Inclusion of 3.5 for / in is 1 st B0
	1 st M1	For a correct method of pooling the classes at both ends [ft their s value]
	2 nd M1	For an attempt at the test statistic, at least 3 correct expressions/values (to awrt/truncated 2 d.p.)
	1 st A1	awrt 8.38 or awrt 8.39 (This implies the both M1 marks)
	2 nd B1ft	For their evaluated $n - 1 - 1$. i.e. realising that they must subtract 2 from their n .
	3 rd B1ft	Correct ft for their $\chi^2_k(0.05)$, where $k = n - 1 - 1$ from their n . (May see 9.488, 12.592, 14.067)
	2 nd A1	Dep. on at least 1 M1 mark for a correct conclusion which is accepting H_0 .
	Note	No follow through on their hypotheses if they are stated the wrong way round.
	Note	Contradictory statements score A0. E.g. "significant, do not reject H_0 "
	Note	Condone the mention of Po(3.5) in conclusion.

Question Number	Scheme	Marks
5. (a)	Label beginners 1 – 452, intermediates 1 – 251, professionals 1 – 97 Use <u>random numbers</u> to select a ... Simple random sample of <u>28 beginners</u> , <u>16 intermediates</u> and <u>6 professionals</u> .	M1 M1 A1 [3]
	(b) Any one of <ul style="list-style-type: none"> Enables estimation of statistics/sampling errors for each strata. Reduces variability. More representative of the population/reflects population structure 	B1 [1]
	(c) $H_0 : m_1 - m_B = 3$ $H_1 : m_1 - m_B > 3$	B1; B1
	$s.e. = \sqrt{\frac{38.1}{60} + \frac{57.3}{80}}$ {= 1.162432794...}	M1
	$z = \frac{36.9 - 31.7 - 3}{"1.1624..."} ; = 1.89258...$	dM1; awrt <u>1.89</u> A1
	One tailed c.v. $Z = 1.6449$ or $CR : Z \geq 1.6449$ or p-value = awrt $0.029 < 0.05$ [in the CR/significant/Reject H_0 /"0.029" < 0.05]	B1
	Conclude either that the <ul style="list-style-type: none"> <u>mean score of intermediates</u> is more than <u>3 greater</u> than the <u>mean score of beginners</u>. (oe) <u>manager's belief is correct</u>. 	A1
		[7]
		11
	Alternative method for "2nd M1, 1st A1, 3rd B1" marks: Let $D = \bar{x}_I - \bar{x}_B$	
$1.6449 = \frac{D - 3}{1.1624...}$	dependent upon the 1st M1 for $\frac{D - 3}{\text{their "1.1624..."}} = 1.6449 / 1.645 / 1.64 / 1.65$	dM1:
So, $D = 4.912...$	$D = \text{awrt } 4.91$ and $D_{\text{obs}} = 5.2$	A1
$D_{\text{obs}} = 36.9 - 31.7 = 5.2$	[1.64, 1.65]	B1

Notes		
(a)	1 st M1 2 nd M1 A1	for a suitable numbered/labelled list for each ability level for use of random numbers/sample to select beginners, intermediates and professionals. (dependent on either the 1st or the 2nd M1 mark) For <u>28 beginners</u> , <u>16 intermediates</u> and <u>6 professionals</u> .
(c)	1 st B1 2 nd B1 Note	$H_0 : m_1 - m_B = 3$ oe $H_1 : m_1 - m_B > 3$ oe If m_1, m_2 used then it must be clear which one refers to intermediates/beginners.
	1 st M1	$s.e. = \sqrt{\frac{38.1}{60} + \frac{57.3}{80}}$. May be implied by s.e. = awrt 1.16 Condone minor slips e.g. $\sqrt{\frac{38.1}{80} + \frac{57.3}{60}}$
	2 nd dM1 1 st A1 3 rd B1 2 nd A1	Dependent upon the 1 st M1. (follow through their s.e. if 1 st M1 mark has been awarded) awrt 1.89 $1.64 \leq C.V. \leq 1.65$ (compatible sign with their test statistic) or a correct probability comparison. Dep. on all M1 and B1 marks scored for contextualised comment which is rejecting H_0 .

Question Number	Scheme		Marks	
6. (a)	$\bar{x} = 230.5$; 95% confidence limits for m are			
	$230.5 \pm 1.96 \cdot \frac{1.2}{\sqrt{5}}$	their $\bar{x} \pm z \cdot \frac{1.2}{\sqrt{5}}$	M1	
			$z = 1.96$	B1
	$= (229.44815\dots, 231.55185\dots) = \text{awrt}(229.4, 231.6)$		At least one end-point is correct.	A1
			Both end-points are correct.	A1
			[4]	
	(b)	{ Let X = number of confidence intervals that <i>don't contain</i> m }		
{ So $X \sim$ } B(20,0.05)				
{ $P(X > 3)$ } = $1 - P(X \leq 3)$ or $1 - 0.9841$				
= 0.0159		awrt 0.0159	A1	
		[3]		
		7		
Notes				
(b)	M1 1st A1 2nd A1	Writing or using either $X \sim B(20, 0.05)$ or $Y \sim B(20, 0.95)$ $1 - P(X \leq 3)$ or $1 - 0.9841$ or $P(Y \leq 16)$. Can be implied by the final answer. awrt 0.0159		

Question Number	Scheme		Marks
7. (a)	$A = \frac{X_1 + X_2 + X_3 + Y_1 + Y_2}{5}$, $X \sim N(30, 4.5^2)$, $Y \sim N(20, 3.5^2)$; X, Y are independent.		
	$E(A) = \frac{3(30) + 2(20)}{5}$ or $\text{Var}(A) = \frac{3(4.5)^2 + 2(3.5)^2}{25}$	A correct method for finding $E(A)$ or $\text{Var}(A)$	M1
	$E(A) = 26$ or $\text{Var}(A) = 3.41$	At least one of either $E(A) = 26$ or $\text{Var}(A) = 3.41$	A1
		Both $E(A) = 26$ and $\text{Var}(A) = 3.41$	A1
	{So $A \sim N(26, 3.41)$ }		
	$\{P(A < 24) = \} P\left(Z < \frac{24 - 26}{\sqrt{3.41}}\right)$	Standardising (\pm) with their mean and their standard deviation	M1
	$= P(Z < -1.08306\dots)$		
	$= 1 - 0.8599$		M1
	$= 0.1401$ (or 0.139391...)		A1
	0.14 or awrt 0.140 or awrt 0.139		
[6]			
(b)	$W \sim N(m, 2.8^2)$; $P(W - X < 4) = 0.1$ W, X are independent.		
	$\{E(W - X) = E(W) - E(X) = m - 30\} \supset E(W - X) = m - 30$	$E(W - X) = m - 30$	B1
	$\{\text{Var}(W - X) = \} 2.8^2 + 4.5^2 \{= 28.09\}$	$2.8^2 + 4.5^2$	M1
	{So $W - X \sim N(m - 30, 28.09)$ }		
	$\{P(W - X < 4) = 0.1\} \Rightarrow P\left(Z < \frac{4 - (m - 30)}{\sqrt{2.8^2 + 4.5^2}}\right) = 0.1$		
	$\frac{4 - (m - 30)}{\sqrt{2.8^2 + 4.5^2}} = k$ ($= -1.2816$)	Standardising (\pm) with their mean which is in terms of m and their standard deviation and setting the result equal to k , where $ k $ is in the interval $[1.28, 1.29]$.	M1
			± 1.2816 or awrt ± 1.2816
			Correct equation . See notes
$\{m = 34 + 1.2816(5.3) \supset\} m = 40.792\dots (= 40.784 \text{ from using } -1.28)$	awrt 40.8	A1	
[6]			
12			
Notes			
(a)	3 rd M1	For a probability tail compatible with 24 and their mean	
(b)	2 nd M1	Allow $\pm \frac{4 - \text{their } E(W - X)}{\sqrt{\text{their } \text{Var}(W - X)}} = k$, where $ k $ is in the interval $[1.28, 1.29]$	
	2 nd B1	For either -1.2816 or 1.2816	
	1 st A1	E.g. Allow $\frac{4 - (m - 30)}{\sqrt{2.8^2 + 4.5^2}} = [-1.29, -1.28]$ or $\frac{(m - 30) - 4}{\sqrt{2.8^2 + 4.5^2}} = [1.28, 1.29]$	

Question Number	Scheme	Marks
8.	X follows a continuous uniform distribution over $[a + 3, 2a + 9]$; $Y = \frac{2\bar{X}}{3} + k$	
(a)	$\{E(\bar{X}) = m\} = \frac{2a + 9 + a + 3}{2}$	M1
	$= \frac{3a}{2} + 6$ or $\frac{3a+12}{2} \neq a$. {So \bar{X} is a biased estimator.}	A1
		[2]
(b)	bias $\left\{ = \frac{3a}{2} + 6 - a \right\} = \frac{1}{2}a + 6$ or $\frac{a+12}{2}$ (allow \pm)	B1ft
		[1]
(c)	$\left\{ E(Y) = \frac{2}{3}E(\bar{X}) + k = a \Rightarrow \right\} \frac{2}{3}\left(\frac{3a}{2} + 6\right) + k = a$	M1
	$\{a + 4 + k = a \Rightarrow\} k = -4$	$k = -4$ A1
		[2]
(d)	$\left\{ \hat{a} = \frac{2}{3}\bar{X} - 4 \Rightarrow \right\} \hat{a} = \frac{2}{3}(7.8) - 4 \{=1.2\}$	M1
	Max value = $2(1.2) + 9$	M1
	$= 11.4$	11.4 or $11\frac{2}{5}$ or $\frac{57}{5}$ A1
		[3]
		8

Notes

(a)	M1	Using the formula $\left(\frac{b+a}{2}\right)$ or obtaining $\frac{3a+12}{2}$ or $\frac{3a}{2} + 6$
	A1	$\frac{3a}{2} + 6$ or $\frac{3a+12}{2}$ and $\neq a$.
(b)	B1ft	bias = $\pm\left(\frac{1}{2}a + 6\right)$ or $\pm\left(\frac{a+12}{2}\right)$ or ft their μ provided $\mu \neq a$
(c)	M1	Sets $\frac{2}{3}(\text{their } E(\bar{X})) + k = a$. This mark can be implied.
	A1	$k = -4$. Note that $k = -4$ with no working is M1 (implied) A1.
(d)	1 st M1	An attempt to use the sample data given to find $\frac{2}{3}\bar{x} + \text{"their } k\text{"}$.
		Allow full expression for \bar{x} or $\frac{\sum x}{n}$. (Note that from the data $\bar{x} = 7.8$)
	2 nd M1	$2 \times \text{"their } a\text{"} + 9$ where their a is a function of the sample mean – which has been found by applying
		$\frac{\sum x}{n}$ from the data values given in the question.
	A1	11.4 cao
	Note	$2(10.6) + 9 = 30.2$ is M0M0A0

